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RADIATIVE HEAT-TRANSFER EFFECTS ON THE PROPAGATION OF PRESSURE SHOCKS

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Abstract—The propagation of a pressure shock has been studied by taking into account the effects of radiative heat transfer. The velocity of propagation of the pressure shock has been determined and its variation with the strength of the shock is computed. It is found that the velocity of propagation of the pressure shock decreases continuously with the decrease of its strength during propagation till the pressure discontinuity vanishes ultimately. The effects of radiative heat transfer will slow down the rate of decrease of the velocity of propagation. A differential equation governing the variations of the strength of the pressure shock during propagation has been determined and solved numerically. The numerical results show that the strength of the pressure shock will continuously decrease till the pressure discontinuity vanishes ultimately. The radiative heat-transfer effects will slow down the rate of decrease in the strength of the pressure shock. The results of Thomas are recovered as a particular case.

NOMENCLATURE

ρ, density;
p, pressure;
T, absolute temperature;
U_i, velocity components;
k, coefficient of thermal conductivity;
K_{eff}, coefficient of effective thermal conductivity;
D_R, Rosseland diffusion coefficient;
a_R, Stefan–Boltzmann constant;
G, velocity of propagation of the pressure shock;
e, internal energy of the gas per unit mass;
C_P, specific heat at constant pressure;
μ, coefficient of viscosity;
Pr, Prandtl number;
R_f, radiative flux number;
R, gas constant;
λ, discontinuity in the velocity gradient;
ξ, discontinuity in the pressure gradient;
ζ, discontinuity in the density gradient;
η, non-dimensional parameter of the strength of the pressure shock;
δ, non-dimensional parameter of the velocity of propagation of the pressure shock;
ν_i, components of the unit normal vector;
M, $= \frac{4}{3}\dfrac{\mu}{\rho_1 R}$;
A, $= 4a_R D_R$.

1. INTRODUCTION

SEVERAL authors [1–4] have made significant contributions in the study of propagation of disturbances in a variety of media. In our recent work [5, 6] we have studied the propagation of sonic discontinuities in radiating gases. The object of the present paper is to study the propagation of a pressure shock by taking into account the effects of viscosity and heat conductivity in flows of radiating gases. In dealing with certain problems like that of blast waves, the effects of pressure and temperature are of primary concern. Since the gas is viscous, we can assume discrete discontinuities in pressure and temperature, while the velocity and density are continuous over the surface of discontinuity. Such a discontinuity is defined as a pressure shock [3]. We shall study the propagation of a pressure shock in a uniform flow of a radiating gas.

When the temperature of the gas is not too high and the gas density is not too low, the radiation energy density and radiation pressure can be neglected [7]. Pai [7] has shown that when the temperature T is below 10^3 K, the radiation terms are negligibly small. When the temperature is in the order of 10^4 K, the heat flux of radiation is of the same order as that of heat transfer by convection and conduction. Near $T = 10^5$ K, the radiation stresses and radiation energy density are no longer negligible and the interaction between the radiation field and the gas-dynamic field are to be taken into account. In the present investigation the temperature of the gas is assumed to be in the order to 10^4 K and Rosseland approximation for a thick gas medium has been used for the radiation energy flux in local thermodynamic equilibrium. Under this approximation the radiant energy flux in local equilibrium is proportional to the temperature gradient and hence the radiative transfer term is similar to heat-conduction term. In this case, the coefficient of effective thermal conductivity is given by [7]

$$K_{\text{eff}} = K + 4D_R a_R T^3 \tag{1.1}$$

where K is the coefficient of thermal conductivity. D_R is the Rosseland diffusion coefficient and a_R is the Stefan–Boltzmann constant.

The following boundary conditions, which are found appropriate to this problem, are assumed in this discussion [3]

$$\bar{U}_i = 0; \tag{1.2a}$$

$$\frac{d\bar{p}}{dn} = 0; \tag{1.2b}$$

$$\frac{d\bar{T}}{dt} = 0 \tag{1.2c}$$

where U_i are the velocity components, p is the pressure and T is the temperature. The bar appearing in relations (1.2) denotes evaluation at the rear or flow side of the shock surface $\sum(t)$. The pressure condition in (1.2) states that the normal directional derivative of the pressure vanishes on the flow side of $\sum(t)$. The temperature condition in (1.2) implies that the temperature of a material particle has a stationary value at the instant of its contact with the rear of the surface $\sum(t)$. In view of the fact that the velocity vanishes on the surface $\sum(t)$, this condition can also be written as

$$\frac{\partial \bar{T}}{\partial t} = 0. \tag{1.3}$$

2. BASIC EQUATIONS

The fundamental equations governing the system are

$$\frac{\partial \rho}{\partial t} + \rho_{,i} U_i + \rho U_{i,i} = 0, \tag{2.1}$$

$$\rho \frac{\partial U_i}{\partial t} + \rho U_j U_{i,j} - \sigma_{ij,j} = 0, \tag{2.2}$$

$$\frac{\partial}{\partial t}\left(\rho e + \frac{\rho}{2} U^2\right) + \frac{\partial}{\partial x_K}\left\{U_K\left(\rho e + \frac{\rho}{2} U^2\right)\right\} - \sigma_{iK}\frac{\partial U_i}{\partial x_K} - \frac{\partial}{\partial x_K}\left(K_{\text{eff}}\frac{\partial T}{\partial x_K}\right) = 0, \tag{2.3}$$

$$p = \rho R T \tag{2.4}$$

where

$$\sigma_{ij} = -p\delta_{ij} - \tfrac{2}{3}\mu U_{K,K}\delta_{ij} + \mu(U_{i,j} + U_{j,i}). \tag{2.5}$$

U_i, p, ρ and e stand for velocity components, pressure, density and internal energy of the gas respectively. The usual summation convention is employed and a comma followed by an index i denotes differentiation with respect to the coordinates x_i of a rectangular system.

The geometrical and kinematical conditions of the first order for a singular surface can be put in the forms [8]:

$$[U_{i,j}] = \bar{U}_{i,j} = \lambda_i \nu_j, \tag{2.6a}$$

$$\left[\frac{\partial U_i}{\partial t}\right] = \frac{\partial \bar{U}_i}{\partial t} = -\lambda_i G, \tag{2.6b}$$

$$[\rho_{,i}] = \bar{\rho}_{,i} = \zeta \nu_i, \tag{2.7a}$$

$$\left[\frac{\partial \rho}{\partial t}\right] = \frac{\partial \bar{\rho}}{\partial t} = -\zeta G, \tag{2.7b}$$

where $\lambda_i = [U_{i,j}]\nu_j$ and $\zeta = [\rho_{,j}]\nu_j$ are functions defined over the surface $\sum(t)$. The bracket $[Z]$ denotes the jump in the quantity enclosed at contiguous points of the singular surface $\sum(t)$. The unit normal to the surface $\sum(t)$ with components ν_i is assumed directed into the upstream region so that the normal velocity G of the surface $\sum(t)$ will have a positive value.

The Rankine–Hugoniot shock relations are

$$\rho(U_n - G) = \bar{\rho}(\bar{U}_n - G), \tag{2.8}$$

$$[\sigma_{ij}]\nu_j = \rho(U_n - G)[U_i], \tag{2.9}$$

$$[\sigma_{ij} U_j]\nu_i + [K_{\text{eff}} T_{,i}]\nu_i = \rho(U_n - G)[E], \tag{2.10}$$

where

$$[E] = C_v[T] + \tfrac{1}{2}[U^2].$$

When we combine (2.5) and (2.9) and make use of the equation (2.6a), we obtain

$$[p]\nu_i + \tfrac{2}{3}\mu\lambda_K \nu_K \nu_i = \mu\lambda_i + \mu\lambda_K \nu_K \nu_i. \tag{2.11}$$

Let us now represent the surface $\sum(t)$ parametrically by functions $x_i(U^1, U^2, t)$ and let us denote by $x_{i\alpha}$ the derivatives of the space coordinates x_i with respect to the parametric coordinates $U^\alpha(\alpha = 1, 2)$. We assume that $\sum(t)$ is regular in the sense that the functional matrix $\|x_{i\alpha}\|$ has rank 2 at points of this surface. The quantities $x_{i\alpha}$ are the components of the projective tensor of the surface $\sum(t)$. In view of (2.11) we can write

$$\lambda_i = \lambda \nu_i \tag{2.12}$$

where λ is a scalar function on the surface $\sum(t)$.

It follows from (2.6a) and (2.12) that the vorticity of the flow field is continuous across the wave surface $\sum(t)$. This shows that a pressure shock is an irrotational wave. From equations (2.11) and (2.12) we get

$$[p] = \tfrac{4}{3}\mu\lambda. \tag{2.13}$$

The relation (2.13) provides us an equation for the discontinuity in the pressure. Also taking jump of relation (2.4) and making use of (2.13) we obtain a corresponding equation for the discontinuity in the temperature in the form

$$[T] = \tfrac{4}{3}\frac{\mu}{\rho_1 R}\lambda. \tag{2.14}$$

Making use of the equations (2.10), (2.13) and (2.14) we obtain

$$[T_{,i}]\nu_i = \frac{-\rho_1 G C_v M\lambda}{K_{1\text{eff}} + A(M^3\lambda^3 + 3T_1 M^2\lambda^2 + 3T_1^2 M\lambda)}, \tag{2.15}$$

where $M = \tfrac{4}{3}(\mu/\rho_1 R)$, $A = 4a_R D_R$ and ρ_1 and T_1 are the constant density and temperature of the gas in front of the shock wave. If we make use of (2.6) and (2.7) in (2.1), we get

$$\zeta G = \rho_1 \lambda. \tag{2.16}$$

In consequence of the boundary conditions (1.2b) and (1.2c) it is now readily seen that the compatibility conditions of the first order for the pressure and temperature can be put in the following forms [8]:

$$[p_{,i}] = \tfrac{4}{3}\mu g^{\alpha\beta}\lambda_{,\alpha} x_{i\beta} \tag{2.17a}$$

$$\left[\frac{\partial p}{\partial t}\right] = \tfrac{4}{3}\mu\frac{\partial \lambda}{\partial t} \tag{2.17b}$$

$$[T_{,i}] = \bar{T}_{,i} = \bar{T}_{,j}\nu_j\nu_i + \tfrac{4}{3}\frac{\mu}{\rho_1 R} g^{\alpha\beta}\lambda_{,\alpha} x_{i\beta} \tag{2.18a}$$

$$\frac{\delta[T]}{\delta t} = G[T_{,i}]\nu_i \tag{2.18b}$$

where $g^{\alpha\beta}$ are the contravarient components of the fundamental metric tensor of the surface $\sum(t)$. The $\lambda_{,\alpha}$ are the surface derivatives and $\delta\lambda/\delta t$ is the δ-time derivative of the scalar function λ.

Using (2.14) and (2.15) in (2.18b), we obtain

$$\frac{\delta[T]}{\delta t} = \frac{-\rho_1 G^2 C_v M\lambda}{K_{1\text{eff}} + A(M^3\lambda^3 + 3T_1 M^2\lambda^2 + 3T_1^2 M\lambda)}. \quad (2.19)$$

Differentiating (2.4) with respect to x_i, we are led to the relations

$$[p_{,i}] = \rho_1 R[T_{,i}] + R[\rho_{,i}T], \quad (2.20)$$

$$[\rho_{,i}T] = [\rho_{,i}][T] + T_1[\rho_{,i}]. \quad (2.21)$$

Multiplying (2.20) by v_i and making use of (2.17a), (2.21), and (2.16) we get

$$G^2 = \frac{3\gamma}{4Pr\rho_1 K}(p_1 + \tfrac{4}{3}\mu\lambda) \times \{K_{1\text{eff}} + A(M^3\lambda^3 + 3T_1 M^2\lambda^2 + 3T_1^2 M\lambda)\}, \quad (2.22)$$

where $Pr = C_P\mu/K$ is the Prandtl number. The second term of the first paranthetical expression in (2.22) is equal to the discontinuity $[p]$ in pressure and the second paranthetical expression in (2.22) is equal to the jump in the radiative transfer term. For a non-radiating gas the relation (2.22) assumes the form

$$G^2 = \frac{3\gamma}{4Pr\rho_1}(p_1 + \tfrac{4}{3}\mu\lambda), \quad (2.23)$$

which is the expression for velocity of propagation of the moving surface $\sum(t)$ derived by Thomas [3]. For weak shocks, i.e. for sufficiently small values of the discontinuity $[p]$, the velocity of the weak shock can be deduced from (2.22) in the form

$$G^2 = \frac{3\gamma p_1}{4Pr\rho_1}(1 + R_f), \quad (2.24)$$

where R_f is the radiation flux number defined as

$$R_f = \frac{4D_R a_R T_1^3}{K}.$$

Rewriting the equation (2.22) in the non-dimensional form, we have

$$\delta^2 = (\eta + 1)\{(1 + R_f) + R_f(\eta^3 + 3\eta^2 + 3\eta)\}, \quad (2.25)$$

where

$$\delta = \frac{G}{C_1}\sqrt{(\tfrac{4}{3}Pr)} \quad \text{and} \quad \eta = \frac{[p]}{p_1}.$$

Here δ is the non-dimensional parameter of the velocity of propagation of the pressure shock and η is the non-dimensional parameter of the strength of the pressure shock. The variations of δ vs η are shown in Fig. 1. The value of δ corresponding to $\eta = 0$ gives the velocity of propagation of a weak shock. It is interesting to note that the velocity of propagation of the pressure shock decreases continuously with the decrease of its strength during propagation till the pressure discontinuity vanishes ultimately. The effects of radiative heat transfer will slow down the rate of decrease of the velocity of propagation.

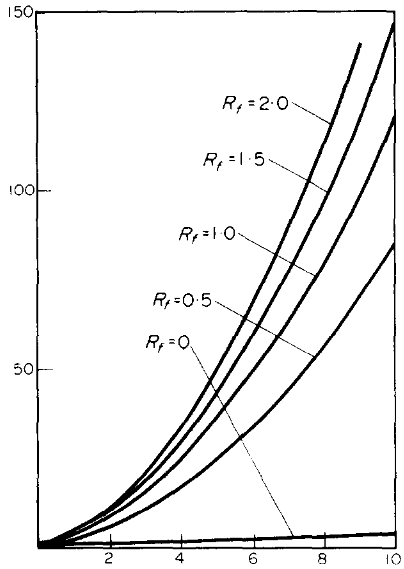

FIG. 1. Variation of the velocity of propagation with respect to the strength of the pressure shock.

3. VARIATION OF THE STRENGTH OF A PRESSURE SHOCK DURING PROPAGATION

Taking the δ-time derivative of (2.14), we have

$$\frac{\delta[T]}{\delta t} = M\frac{\delta\lambda}{\delta t}. \quad (3.1)$$

Using (2.19) and (3.1), it follows that

$$\frac{\delta\lambda}{\delta t} = \frac{-\rho G^2 C_v \lambda}{K_{1\text{eff}} + A(M^3\lambda^3 + 3T_1 M^2\lambda^2 + 3T_1^2 M\lambda)}. \quad (3.2)$$

Substituting for G^2 from (2.22) in (3.2), we have

$$\frac{\delta\lambda}{\delta t} = -\frac{3\gamma C_v}{4PrK}(p_1 + \tfrac{4}{3}\mu\lambda)\lambda,$$

which can be put in the form

$$\frac{\delta\eta}{\delta t} = \frac{-3p_1}{4\mu}\eta(\eta + 1). \quad (3.3)$$

Let $\sum(t_0)$ represent the position of the wave surface at time $t = t_0$ and let σ denote the distance measured from $\sum(t_0)$ along the normal trajectories to the family of surfaces $\sum(t)$ in the direction of propagation. The discontinuity η can be regarded as a function of the distance σ along the normal trajectory and hence we have

$$\frac{\delta\eta}{\delta t} = G\frac{d\eta}{d\sigma}. \quad (3.5)$$

Making use of (3.5) and (2.22) in (3.3) we get

$$\frac{d\eta}{d\Gamma} = \frac{-\eta(1+\eta)^{\frac{1}{2}}}{\{1 + R_f + R_f(\eta^3 + 3\eta^2 + 3\eta)\}^{\frac{1}{2}}}, \quad (3.6)$$

where

$$\Gamma = \frac{p_1}{C_1\mu}\sigma\sqrt{(\tfrac{4}{3}Pr)}$$

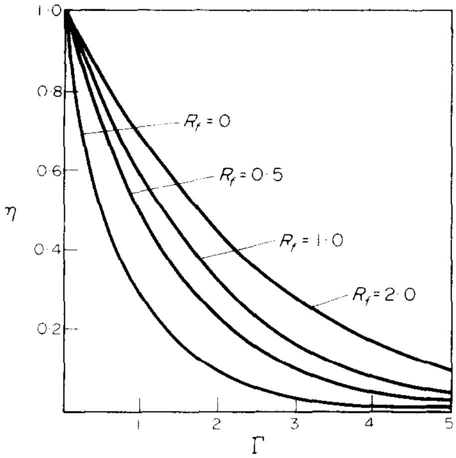

FIG. 2. Variation of the strength of the pressure shock during propagation.

When the radiative transfer term is neglected, equation (3.6) takes the form

$$\frac{d\eta}{d\Gamma} = -\eta(\eta+1)^{\frac{1}{2}}. \tag{3.7}$$

Solving (3.7), we get the solution for η in the form

$$(\eta+1)^{\frac{1}{2}} = \frac{1+Be^{-\Gamma}}{1-Be^{-\Gamma}}, \tag{3.8}$$

where B is the constant of integration. If we suppose that the initial pressure discontinuity $\eta_0 = 1$ at $\Gamma = 0$, the solution (3.8) can be put in the form

$$\eta = \frac{4}{(3+2\sqrt{2})e^{\Gamma}+(3-2\sqrt{2})e^{-\Gamma}-2}, \tag{3.9}$$

which shows that $\eta \to 0$ as $\Gamma \to \infty$. This is in full agreement with the conclusion of Thomas [3].

The variations of the strength of the pressure shock during propagation are exhibited in Fig. 2. The strength of the pressure shock decreases continuously till the discontinuity vanishes ultimately. The effects of radiative heat transfer will slow down the rate of decrease of the strength of the pressure shock during propagation.

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EFFET DU TRANSFERT PAR RAYONNEMENT SUR LA PROPAGATION DES ONDES DE CHOC

Résumé—La propagation d'un choc de pression a été étudiée en tenant compte des effets du transfert par rayonnement. La vitesse de propagation a été déterminée et sa variation avec l'intensité du choc ont été calculées. On a trouvé que la vitesse de propagation décroit continûment avec son intensité pendant la propagation jusqu'à la disparition finale de la discontinuité de pression. L'effet du transfert de chaleur par rayonnement ralentit la décroissance de la vitesse de propagation. Une équation différentielle qui gouverne les variations de l'intensité du choc de pression pendant sa propagation a été déterminée et résolue numériquement. Les résultats numériques montrent que l'intensité du saut de pression décroit continûment jusqu'à la disparition finale de la discontinuité de pression. L'effet du transfert de chaleur par rayonnement diminue le taux de décroissance de l'intensité du saut de pression. Les résultats de Thomas sont restrouvés comme un cas particulier.

EINFLUSS DES STRAHLUNGS-WARMEUBERGANGS AUF DIE FORTPFLANZUNG VON VERDICHTUNGSSTOSSEN

Zusammenfassung—Die Fortpflanzung eines Verdichtungsstosses wurde untersucht unter Berücksichtigung der Einflüsse des Strahlungs-Wärmeüberganges. Die Fortpflanzungsgeschwindigkeit wurde bestimmt und ihre Änderung mit der Stärke des Stosses berechnet. Es zeigte sich, dass die Fortpflanzungsgeschwindigkeit kontinuierlich abnimmt, mit abnehmender Druckstärke, bis schliesslich der Drucksprung verschwindet. Der Einfluss des Strahlungswärmeüberganges verringert diese Abnahme der Fortpflanzungsgeschwindigkeit. Eine Differentialgleichung, die die Änderung der Stärke des Verdichtungsstosses während seiner Fortpflanzung charakterisiert wurde aufgestellt und numerisch gelöst. Die numerischen Ergebnisse zeigen, dass die Stärke des Verdichtungsstosses kontinuierlich abnimmt, bis der Drucksprung schliesslich verschwindet. Die Ergebnisse von Thomas werden als ein spezieller Fall erhalten.

ВЛИЯНИЕ ЛУЧИСТОГО ПЕРЕНОСА НА РАСПРОСТРАНЕНИЕ СКАЧКОВ УПЛОТНЕНИЯ

Аннотация — Изучался процесс распространения скачка уплотнения с учетом влияния лучистого переноса. Определялась скорость распространения ударной волны, и учитывалось её изменение с изменением интенсивности ударной волны. Найдено, что при распространении ударной волны скорость её распространения постоянно уменьшается до тех пор, пока скачок давления окончательно не исчезнет. Лучистый перенос тепла снижает темп уменьшения скорости распространения ударной волны. Выведено и численно решено дифференциальное уравнение, определяющее изменение интенсивности ударной волны во время её распространения. Численные результаты показывают, что интенсивность ударной волны постоянно уменьшается до исчезновения скачка давления. Лучистый перенос тепла замедляет темп снижения интенсивности ударной волны. Как частный случай приводятся результаты Томаса.